A DESCRIPTION OF EFFECT OF COMPOSITION OF MIXED BINARY SOLVENTS ON PROCESSES IN SOLUTIONS

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A theoretical description of the effect of changed composition of mixed solvents on processes in solutions has been suggested on the basis of the proportionality between the Gibbs energy change of the process and that of the solvent due to the transition from pure components to the mixture. The additional Gibbs energy has been expressed by means of the so-called classical functions by Margules, van Laar-Wohl, and van Laar-Null. The application to 115 various processes (pK, IR, UV-VIS, NMR, log k, and others) has confirmed that the theoretical presumptions are justified, the most suitable being Margules' 4th order model which shows a statistically significant difference from the models of lower orders.

Although from practical point of view solvent mixtures are more important than one-component solvents, the effects of solvents mixtures on various processes in solutions are less investigated. This fact is obviously caused by greater complexity and diversity of the problem, as in addition to the effect itself (i.e. that of the solvent on the process) mutual interactions of the solvent components are also important, which leads to non-additivity of the effect and behaviour of the mixture.

The most frequent approach to evaluation of effects of mixed solvents on processes in solutions is based on applications of functions of relative permittivity and refractive index (e.g. $refs^{1-9}$). The drawbacks of these models are revealed by measurements in "isodielectric" mixtures^{10,11} which indicate further types of interactions. One of the approaches even applies the description of the dependence of logarithm of rate constant on logarithm of water concentration in hydrolytical reactions¹²⁻¹⁸, which is an often misleading oversimplification. A more successful application of this approach is based on the behaviour (of rate constants in this case) in one-component solvents which, according to the ideas about the existence of solvates, is decisive for a prediction of the behaviour in mixtures¹⁹⁻²¹. Empirical procedures based on applications of extrathermodynamic relations already have their tradition based on the Y parameter²². This concept was developed later with application of other standard substrates^{8,9,23-29}. The applicability of these parameters was increased by the suitable transformation³⁰ which leads to a linear dependence of the result of a process on the empirical parameters within the whole concentration range of the

components in a mixed solvent. While the choice of the standard model is critical for these empirical relations, the empirical relations for multicomponent mixtures suffer from an additional disadvantage consisting in the vast number of combinations of components and compositions which can be neither measured nor tabulated. From this point of view more practical are the ways of interpretation based on the properties of the pure components with the help of relations which can suitably approximate the behaviour of solutes depending on the solvent composition. Although these are often non-linear dependences with several parameters (which need a greater number of experimental points), they are advantageous in being more general and allowing the interpretation of the parameter values obtained.

The aim of this work is to suggest and verify a relation for description of effects of mixed solvents on processes in solutions on the basis of physico-chemical ideas with application of the expansions for the additional Gibbs' function.

THEORETICAL

Any physical or chemical process in a solution can be described as a function of the Gibbs energy difference between the initial and final states. As there always are substantially less molecules undergoing a process in the solution than the solvent molecules, the effect of the process on the solvent as a whole can be - in the first approximation - neglected. If the process taking place in the solution is taken as one system and the solvent as the other system, then the difference between the chemical potentials "after the process" and "before the process" can be considered proportional to that of solvent, α being the coefficient of proportionality (sensitivity)

$$\mu^{\text{after}} - \mu^{\text{before}} = \alpha \cdot \mu^{\text{solv.}} . \tag{1}$$

According to the definition the chemical potential is the change of the Gibbs energy with the change of the number of mol at constant pressure and temperature (the superscripts p and s denote the process and solvent, respectively):

$$\partial \Delta G^{\mathbf{p}}/\partial n_i = \alpha (\partial G^{\mathbf{s}}/\partial n_i); \quad p, T = \text{const.}; \quad n_{i \neq j},$$
(2)

hence also

$$d\Delta G^{p} = \alpha \, dG^{s}; \quad p, T = \text{const.}$$
 (3)

Expressing the total differentials as functions of mol numbers we obtain Eq. (4), where q means number of components.

$$\sum_{i=1}^{q} (\partial \Delta G^{\mathbf{p}} / \partial n_i) \, \mathrm{d}n_i = \sum_{i=1}^{q} (\partial G^{\mathbf{s}} / \partial n_i) \, \mathrm{d}n_i \, ; \quad p, T = \text{const.} \, ; \quad n_{i \neq j} \, . \tag{4}$$

The integration in the sense from pure components to a real mixture gives Eq. (5),

$$\Delta\Delta G^{\mathbf{p}} = \sum_{i=1}^{q} n_i \,\Delta\Delta G^{\mathbf{p}}_i = \alpha \sum_{i=1}^{q} n_i \,\Delta G^{\mathbf{s}}_i = \alpha \,\Delta G^{\mathbf{s}} \,, \tag{5}$$

where $\Delta\Delta G_i^p$ represents the change of ΔG of the process with the change of number of mol on going from the *i*-th pure solvent to the real mixture, and ΔG_i^s means the contribution of the *i*-th solvent to the Gibbs energy change of formation of the mixture from pure components. If the system is ideal (denoted by asterisk), Eq. (6) can be similarly written

$$\Delta\Delta G^{*\mathbf{p}} = \sum_{i=1}^{q} n_i \,\Delta\Delta G_i^{*\mathbf{p}} = \alpha \sum_{i=1}^{q} n_i \,\Delta G_i^{*\mathbf{s}} + \alpha \,\Delta G^{*\mathbf{s}} \,. \tag{6}$$

With application of Eqs (5) and (6), the difference between $\Delta G^{\rm p}$ in a real solution and that in an ideal one can be expressed by Eq. (7) in which $\Delta G^{\rm E}$ is the so-called additional change of Gibbs' function

$$\Delta\Delta G^{\mathbf{p}} - \Delta\Delta G^{*\mathbf{p}} = \alpha (\Delta G^{*} - \Delta G^{*s}) = \alpha \Delta G^{\mathbf{E}}.$$
⁽⁷⁾

If the solution is an ideal one, it can be anticipated that the effects of the individual components of the solvent on the process taking place therein will be identical with those of the same components in the respective one-component solvents. Then Eq. (8) is valid, and Eq. (7) can be rewritten to the form of Eq. (9) in which ΔG_{01}^{p} are ΔG values in pure components

$$\Delta \Delta G^{*p} = 0 \tag{8}$$

$$\Delta G^{\mathbf{p}} = \Delta G_0^{\mathbf{p}} + \alpha \, \Delta G^{\mathbf{E}} = \sum_{i=1}^q n_i \, \Delta G_{0i}^{\mathbf{p}} + \alpha \, \Delta G^{\mathbf{E}} \,. \tag{9}$$

The ΔG^{p} and ΔG_{0i}^{p} values can be expressed by means of ln K of an equilibrium process, ln k of a rate process, or some other analogous quantity with the energy/mol dimension.

The additional change of Gibb's function with the composition of liquid phase can be expressed by Wohl's expansion³¹ which generalizes a number of equations (Margules³², Van Laar³³, Null³⁴, Redlich-Kister³⁵, and others). Beside these so--called classical equations there are non-classical ones.

In terms of the classical equations and with reference to the overall mol number, Eq. (9) for a binary mixture can be rewritten to the following forms (the superscript for "process" is omitted): the application of the regular³² for expressing the additional Gibbs function leads to Eq. (10), the expansions of the 3rd and 4th orders (Margules³²) lead to Eqs (11) and (12), respectively, application of the expansion by van Laar & Wohl³³ gives Eq. (13), whereas that by van Laar and Null³⁴ leads to Eq. (14).

$$\Delta G = x_1 \, \Delta G_{10} + x_2 \, \Delta G_{20} + x_1 x_2 A_{12} \tag{10}$$

$$\Delta G = x_1 \, \Delta G_{10} + x_2 \, \Delta G_{20} + x_1 x_2 (x_1 A_{21} + x_2 A_{12}) \tag{11}$$

$$\Delta G = x_1 \Delta G_{10} + x_2 \Delta G_{20} + x_1 x_2 (x_1 A_{21} + x_2 A_{12} - x_1 x_2 D_{12})$$
(12)

$$\Delta G = x_1 \, \Delta G_{10} + x_2 \, \Delta G_{20} + x_1 x_2 A_{12} A_{21} / (x_1 A_{12} + x_2 A_{21}) \tag{13}$$

$$\Delta G = x_1 \Delta G_{10} + x_2 \Delta G_{20} + x_1 x_2 A_{12} A_{21} (x_1 A_{12} + x_2 A_{21}) / / (x_1 |A_{12}| + x_2 |A_{21}|)^2$$
(14)

The 3rd order equation by Redlich and Kister³⁵ is identical with that by Margules (11). In all the equations the proportionality coefficient is included in the parameters.

RESULTS AND DISCUSSION

For testing of applicability of Eqs (10) through (14) to the description of effects of binary mixed solvents on processes in solutions we used 115 examples from literature (Table I). The testing includes data on dissociation constants, NMR, IR and UV-VIS spectra, rate constants, thermodynamical, electrochemical and other quantities. The optimization¹⁰⁴ of the parameters in the models (10) through (14) applied to the data of Table I gave the residual mean squares which are summarized in Table II for the four-parameter Eqs (11), (13), (14) and the five-parameter Eq. (12). As the data used were of various types and experimental qualities, the differences between the models were tested by means of the Friedman method¹⁰⁵. The testing criterion is defined by Eq. (15),

$$Q = 12 \sum_{j=1}^{k} \left(\sum_{i=1}^{N} \overline{R}_{ij} \right)^2 / (Nk(k+1)) - 3N(k+1), \qquad (15)$$

where k means number of the models tested, N means number of the examples tested, and R_{ij} means the order of the *j*-th model within the *i*-th example tested. If the hypothesis is valid, the Q criterion for $N \to \infty$ asymptotically involves $\chi^2(k-1)$ distributions. The application of the criterion (15) to the data of Table II and values of residual mean square (they are not given for the regular (Eq. (10)) because of the small significance of this model) gave the value Q = 109.89. This value by far exceeds the critical value $\chi^2_{0.05}(4) = 9.49$, wherefrom it follows that the individual methods differ in the value of residual mean square and, hence, in the quality of interpretation of experimental data. The individual methods can easily be compared according to the average order \overline{R} : the Margules 4th order equation (Eq. (12)) 1.86, the van Laar and Null equation (Eq. (14)) 2.89, the Margules 3rd order equation (Eq. (11)) 2.94, the van Laar & Wohl equation (Eq. (13)) 3.33, and the regular (Eq. (10)) 3.98. Hence, it can be stated that addition of one parameter will improve the order by

	Effect	of	Mixed	Binary	Solvents
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The processes in mixed binary solvents used for testing the physical models

No.	Substrate, process	Mixture ^a	References
	Acid-base	equilibria, pK	
1	Benzoic acid, 25°C	MeOH-W	29, 36-50
2	· · · · ·	EtOH-W	29, 40, 50-62
3		DMF-W	63-70
4		DMSO-W	63-65, 71-80
5		Ac–W	68, 70, 71, 81, 82
6		$Ac-W^b$	68, 71, 81, 82
7	Acetic acid, 25°C	MeOH-W	29, 40, 47, 49, 51
8		EtOH-W	29, 40, 51, 52, 62, 83
9		DMF-W	63-65, 68, 69, 84, 85
10		DMSO-W	63-65, 72, 80, 84, 86, 87
11		Ac–W	51, 68, 70, 88
12		$Ac-W^b$	51, 68, 88
13	Formic acid, 25°C	MeOH-W	29, 40, 49
14		EtOH-W	29, 51, 62
15		DMF-W	29, 69
16	Phenol, 25°C	MeOH-W	29
17		EtOH-W	29
18		DMSO-W	63-65, 80
19	Benzenesulphonamide, 25°C	MeOH-W	89
20	· ,	DMF-W	89
21	Oxalic acid, 25°C	EtOH-W	29
22	Malonic acid, 25°C	EtOH-W	29
23	Anion of malon. acid, 25°C	EtOH-W	29
24	Succinic acid, 25°C	EtOH-W	29
25	Anion of succ. acid, 25°C	EtOH-W	29
26	Glutaric acid, 25°C	EtOH-W	29
27	Anion of glut. acid, 25°C	EtOH-W	29
28	Water, 30°C	DMSO-W	90
29	Water, 25°C	DMSO-W	90
30	Water, 20°C	DMSO-W	90
	The complex sta	bility constant $p\beta_{11}$	
31	13	DMSO-W	91
	Spectral	behaviour	
32	The B_{KT} parameter characterizing the solvent basicity	FA-MeOH	24
33	-	Et ₂ O-MeOH	24
		-	

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(Continued)

No.	Substrate, process	Mixture ^a	References
35		AN-MeOH	24
36		Ac-MeOH	24
37		DMF-MeOH	24
38		EA-MeOH	24
39		CHCl ₃ -MeOH	24
40		B-MeOH	24
41		D-MeOH	24
42		W-MeOH	24
43		NM-MeOH	24
44		PC-MeOH	24
45		THF-MeOH	24
46		MeOH-W	8
47		EtOH-W	8
48		1-PrOH–W	8
49		2-PrOH-W	8
50		AN-W	8
51		DMSO-W	8
52		DMF-W	8
53		Ac-W	8
54		THF-W	8
55		D-W	8
56		EtOH-Hx	8
57	The $E_{\rm T}(30)$ parameter characterizing the solvent	MeOH-W	8
58		EtOH-W	8
59		1-PrOH-W	8
60		2-PrOH-W	8
61		AN-W	92
62		DMSO-W	9
63		DMF-W	8
64		Ac-W	8
65		THF-W	8
66		D-W	8
67		EtOH-Hx	92
68	Z-Z(EtOH) shift of abs. maximum	EtOH-W	93
69		MeOH-W	93
70		Ac-W	93
71		D-W	93
72	Solvatochromic shift	DMSO-CCl ₄	94
73		DMSO-PhCl	94

Effect of	Mixed	Binary	Solvents
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(Continued)

No.	Substrate, process	Mixture ^a	References	
74		DMSO-DCE	94	
75		DMSO-DCM	94	
76		THF-CCl4	94	
77		MeOH-CCl ₄	94	
78		t-BuOH-CCl ₄	94	
79	NMR chem. shift ⁵⁹ Co	CHCl3-CCl4	95	
80	$-\delta^{1}H(H_{2}O) +$	AN-W ^c	96	
	δ^1 H((CH ₃) ₃ COH)			
81		$AN-W^{c}$	96	
82		$AN-W^{c}$	96	
83		$AN-W^{c}$	96	
84		$AN-W^{c}$	96	
85	$-\delta^{1}$ H(CH ₃ CN) +	AN-W ^c	96	
	δ^1 H((CH ₃) ₃ COH)			
86		$AN-W^{c}$	96	
87		AN-W ^c	96	
88		$AN-W^{c}$	96	
	Kinetic bel	naviour, log k		
80	\mathbf{Y} (CH.) CC	FtOH_W	97	
91	$(C_1H_2)_3 = C_1H_2I_2$	cHx-Ac	4	
92	334 K	cHx-Ac	4	
93	323 K	cHx-Ac	4	
94	$(C_2H_{\epsilon})_2N + C_2H_{\epsilon}I_2$ 314 K	cHx-Ac	4	
95	$C_6H_5O_2OCH(CH_3)_2$, hydrolysis	Ac-W	12	
96	p-NO ₂ C ₂ H ₂ SO ₂ CH ₂ OClO ₂	tert-BuOH-W	98	
20	hydrolysis		20	
97	$p-CH_3OC_6H_4COCF_3$,	sulfolan-W	99	
	hydratation			
98	$p-CH_3OC_6H_4COCF_3$,	sulfolan-W	99	
	hydratation			
	Thermodynamical char	racteristics, neutraliza	ation	
99	ΔG , benzoic acid	MeOH-W	100	

99	ΔG , benzoic acid	MeOH-W	100
100	ΔH	MeOH-W	100
101	ΔS	MeOH-W	100
102	ΔG	Ac–W	100
103	ΔH	Ac–W	100
104	ΔS	Ac-W	100
105	ΔG , acetic acid	Ac-W	100

(Continued)

No.	Substrate, process	Mixture ^a	References
106	ΔΗ	Ac-W	100
107	ΔS	Ac-W	100
108	ΔG	MeOH-W	100
109	ΔH	MeOH-W	100
110	ΔS	MeOH-W	100
	(Other processes	
111	Conductivity, HCl	DMSO-W	101
112	Viscosity	DMSO-W	101
113	Diffusion coefficient	MeOH-W	102
114	Diffusion coefficient	MeOH-W	102
115	Solubility, NaNO ₂	HNO ₃ -W	103

^a W water, MeOH methanol, EtOH ethanol, DMF dimethylformamide, DMSO dimethyl sulphoxide, Ac acetone, FA formamide, Et_2O diethyl ether, AN acetonitrile, EA ethyl acetate, B benzene, D 1,4-dioxane, NM nitromethane, PC propylene carbonate, PrOH propanol, PhCl chlorobenzene, DCE 1,2-dichloroethane, DCM dichloromethane, tert-BuOH 2-methyl-2-propanol, cHx cyclohexane; ^b without the pK value in pure acetone; ^c at various concentrations of the added AgNO₃.

about 1. If the regular (Eq. (10)) is excluded as a little satisfactory model and the testing is repeated by means of the criterion (15), the value Q = 52.47 is obtained. In this case, too, the critical value $\chi^2_{0.05}(3) = 7.81$ is exceeded, hence the models are not comparable. As it is seen from the newly calculated average order \overline{R} : the Margules 4th order equation (12) 1.77, the van Laar and Null equation (14) 2.57, the Margules 3rd order equation (11) 2.74, and the van Laar and Wohl equation (13) 2.91, the non-equivalence of the equations is predominantly due to the difference between the four- and five-parameter equations. Hence, the fifth parameter brings a significant improvement of the interpretation of experimental data.

Table III present a survey of applicability of the individual model equations (11) through (14) to various data types. It can be seen that introduction of a further parameter (i.e. application of Eq. (12)) is significant for the descriptions of effects of binary solvents on the dissociation, IR, and UV-VIS spectra, being insignificant for NMR spectra and rate constants of chemical reactions. This result is also reflected in the average orders. As, theoretically, the process type is irrelevant, the result found can be interpreted as a low sensitivity of the given process to the composition changes of binary solvents. Small deviations from ideal behaviour are negligible (within

Effect o	fΝ	lixed	Binary	Solvents

Residual mean squares of the models (11) through (14) for the data of Table I

No.	N	Eq. (11)	Eq. (12)	Eq. (13)	Eq. (14)
1	50	$3.922.10^{-2}$	$1.807.10^{-2}$	$1.899.10^{-2}$	$1.899.10^{-2}$
2	38	$6.121.10^{-2}$	$4.129.10^{-2}$	$2.783.10^{-1}$	$8.498.10^{-2}$
3	13	$5.014 \cdot 10^{-2}$	$5.530.10^{-2}$	$9.588.10^{-2}$	$9.796.10^{-2}$
4	24	$1.140.10^{-2}$	$8.492.10^{-3}$	$1.125.10^{-2}$	$1.083 \cdot 10^{-2}$
5	20	$1.798 \cdot 10^{-2}$	$1.582.10^{-2}$	$2.927.10^{-2}$	$2.927.10^{-2}$
6	19	$1.602 \cdot 10^{-2}$	$1.692.10^{-2}$	$2.834.10^{-2}$	$1.653.10^{-2}$
7	40	$4.233 \cdot 10^{-2}$	$1.526.10^{-2}$	$2.108 \cdot 10^{-1}$	$1.732.10^{-2}$
8	41	$1.735 \cdot 10^{-2}$	$6.525 \cdot 10^{-3}$	$1.105 \cdot 10^{-2}$	$1.610.10^{-2}$
9	15	$1.609.10^{-1}$	$1.764.10^{-1}$	$1.606 \cdot 10^{-1}$	$1.631.10^{-1}$
10	23	$4.684 \cdot 10^{-3}$	$1.485.10^{-3}$	$4.178.10^{-3}$	$4.178.10^{-3}$
11	22	$1.374.10^{-1}$	$6.516.10^{-2}$	1·436 . 10 ⁰	$5.604.10^{-1}$
12	21	$4.452.10^{-2}$	$4.730 \cdot 10^{-2}$	$4.415.10^{-2}$	$4.448.10^{-2}$
13	23	$4.265.10^{-2}$	$2.889.10^{-2}$	$2.534.10^{-1}$	$1.308.10^{-1}$
14	19	$2.716.10^{-2}$	$6.225 \cdot 10^{-3}$	$3.283 \cdot 10^{-3}$	$3.283 \cdot 10^{-3}$
15	9	$6.907.10^{-1}$	$8.348.10^{-1}$	$8.440.10^{-1}$	$8.440.10^{-1}$
16	15	$6.000 \cdot 10^{-2}$	$2.385.10^{-2}$	$2.699.10^{-2}$	$2.093.10^{-2}$
17	16	$8.259 \cdot 10^{-2}$	$4.657.10^{-2}$	$6.243.10^{-2}$	$6.243.10^{-2}$
18	13	$2.099 \cdot 10^{-2}$	$1.896.10^{-2}$	$1.494.10^{-2}$	$1.494.10^{-2}$
19	23	$4.203 \cdot 10^{-2}$	$3.069 \cdot 10^{-3}$	$3.432.10^{-1}$	$2.078.10^{-3}$
20	18	$9.757.10^{-3}$	$7.105.10^{-3}$	$5.736.10^{-2}$	$2.629.10^{-2}$
21	10	$2.282 \cdot 10^{-2}$	1·991 . 10 ⁻²	$2.047 \cdot 10^{-2}$	$2.047.10^{-2}$
22	10	$2.702 \cdot 10^{-3}$	$1.600 \cdot 10^{-3}$	$2 \cdot 113 \cdot 10^{-3}$	$2.183.10^{-3}$
23	10	$8.838 \cdot 10^{-2}$	$1.060 \cdot 10^{-1}$	$9.825 \cdot 10^{-2}$	$9.825.10^{-2}$
24	13	$2.447.10^{-2}$	$4.906 \cdot 10^{-3}$	$1.127.10^{-2}$	$1.127.10^{-2}$
25	10	$7.733.10^{-2}$	$9.268 \cdot 10^{-2}$	$8.350.10^{-2}$	$8.350.10^{-2}$
26	11	$2.349.10^{-2}$	$1.690.10^{-2}$	$2.415.10^{-2}$	$2.415.10^{-2}$
27	11	$5.062.10^{-2}$	$5.029 \cdot 10^{-2}$	$5.225.10^{-2}$	$5.225.10^{-2}$
28	21	$2.362.10^{-3}$	$1.898 \cdot 10^{-3}$	$4.426.10^{-3}$	$7.478.10^{-3}$
29	21	$2.504 \cdot 10^{-3}$	$2.023 \cdot 10^{-3}$	$3.728 \cdot 10^{-3}$	$2.714.10^{-3}$
30	21	$3.225.10^{-3}$	$2.834 \cdot 10^{-3}$	$3.719.10^{-1}$	$3.719.10^{-2}$
31	12	$3.794.10^{-3}$	$1.764 \cdot 10^{-3}$	$1.296.10^{-1}$	$1.530.10^{-2}$
32	11	$2.123 \cdot 10^{-5}$	$1.165.10^{-5}$	$1.427.10^{-4}$	$5.432.10^{-5}$
33	11	$6.718.10^{-5}$	$6.992.10^{-6}$	$4.958.10^{-5}$	$4.958.10^{-5}$
34	11	$2.797.10^{-5}$	$4.429.10^{-6}$	$4.868.10^{-5}$	$4.868.10^{-5}$
35	11	$1.069 \cdot 10^{-5}$	$7.751.10^{-6}$	$9.993 \cdot 10^{-5}$	$3.308 \cdot 10^{-5}$
36	11	$1.054 \cdot 10^{-5}$	$1.084 \cdot 10^{-5}$	$3.567.10^{-5}$	$3.567.10^{-5}$
37	11	$4.128.10^{-6}$	$4.759.10^{-6}$	$5.035.10^{-6}$	$5.035.10^{-6}$
38	11	$3.281 \cdot 10^{-5}$	$1.259.10^{-5}$	$3.308.10^{-5}$	$3.308.10^{-5}$
39	11	$6.902.10^{-4}$	$1.870.10^{-4}$	$6.968.10^{-4}$	$6.968.10^{-4}$
40	11	$8.417.10^{-4}$	$4.224.10^{-4}$	$1.171.10^{-4}$	$1.771.10^{-4}$
41	11	$2.723 \cdot 10^{-4}$	$7.907.10^{-5}$	$1.390.10^{-5}$	$1.390.10^{-5}$

No.	N	Eq. (11)	Eq. (12)	Eq. (13)	Eq. (<i>14</i>)
42	11	3·291 . 10 ⁻⁴	2·011 . 10 ⁻⁴	$3.553.10^{-4}$	$3.553.10^{-4}$
43	11	$2.843.10^{-5}$	$2.611 \cdot 10^{-5}$	$1.068.10^{-3}$	$3.196.10^{-5}$
44	11	$2.363 \cdot 10^{-5}$	$1.265.10^{-5}$	$5.403.10^{-4}$	$2.745.10^{-5}$
45	11	$4.046.10^{-5}$	$1.638.10^{-5}$	$1.337.10^{-5}$	$1.337.10^{-5}$
46	11	$3.944.10^{-4}$	$2.063.10^{-4}$	$5.602.10^{-4}$	$5.602.10^{-4}$
47	11	$6.668.10^{-4}$	$6.798.10^{-4}$	$8.753.10^{-4}$	8·753 . 10 ⁻⁴
48	11	$2 \cdot 272 \cdot 10^{-3}$	$1.142.10^{-3}$	$2.697.10^{-4}$	$2.697.10^{-4}$
49	11	$1.790.10^{-3}$	$1.997.10^{-4}$	$2.103.10^{-4}$	$2 \cdot 103 \cdot 10^{-4}$
50	11	$8.302.10^{-4}$	$1.709 \cdot 10^{-4}$	$3.449.10^{-4}$	$3.449.10^{-4}$
51	11	$6.411.10^{-4}$	$9.324.10^{-5}$	$5.142.10^{-4}$	$5.142.10^{-4}$
52	11	$3.719.10^{-4}$	$2.171.10^{-4}$	$8.083.10^{-5}$	$8.083.10^{-5}$
53	11	$6.540.10^{-4}$	$4.935.10^{-4}$	$1.642.10^{-4}$	$1.642.10^{-4}$
54	11	$5.593 \cdot 10^{-3}$	$2.339.10^{-3}$	$3.218 \cdot 10^{-3}$	$5.404.10^{-3}$
55	11	$6.548.10^{-4}$	$3.023.10^{-4}$	$1.080 \cdot 10^{-3}$	$5.588.10^{-4}$
56	12	$3.003 \cdot 10^{-3}$	$4.038 \cdot 10^{-4}$	$5.477.10^{-4}$	$5.477.10^{-4}$
57	11	3·674 . 10 ⁻⁵	$1.204 \cdot 10^{-5}$	$1.080 \cdot 10^{-3}$	$2.239.10^{-5}$
58	11	$2.176.10^{-4}$	$2.261 \cdot 10^{-5}$	$3.281.10^{-3}$	$1.271.10^{-3}$
59	11	$1.650.10^{-3}$	$6.664 \cdot 10^{-4}$	$1.163.10^{-5}$	$1.117.10^{-5}$
60	11	$1.004 \cdot 10^{-3}$	$2.310.10^{-4}$	$5.095 \cdot 10^{-5}$	$4.424.10^{-5}$
61	11	$3.840.10^{-4}$	$2.558.10^{-4}$	$1.161 \cdot 10^{-3}$	$1.161.10^{-3}$
62	11	$1.031.10^{-4}$	$3.170.10^{-5}$	$3.143.10^{-3}$	$6.512.10^{-5}$
63	11	$2.238.10^{-4}$	$3.714.10^{-5}$	$3.377.10^{-5}$	$2.550.10^{-5}$
64	11	$3.352.10^{-3}$	$8.905 \cdot 10^{-5}$	$1.093 \cdot 10^{-3}$	$1.093.10^{-3}$
65	11	$1.223.10^{-3}$	$7.348.10^{-4}$	$6.796.10^{-3}$	$2.744.10^{-3}$
66	11	$7.361.10^{-4}$	$2.644.10^{-4}$	$1.007 \cdot 10^{-3}$	$1.007.10^{-3}$
67	13	$4.534.10^{-3}$	$2.767.10^{-3}$	$7.778.10^{-5}$	$7.778.10^{-5}$
68	17	$1.255.10^{-4}$	$1.326.10^{-4}$	$1.358.10^{-4}$	$1.240.10^{-4}$
69	17	$3.637.10^{-5}$	$2.523 \cdot 10^{-5}$	$4.649.10^{-5}$	$3.564.10^{-5}$
70	19	$3.198.10^{-5}$	$4.131.10^{-6}$	$4.826.10^{-6}$	$4.826.10^{-6}$
71	13	$1.663 \cdot 10^{-5}$	$8.928 \cdot 10^{-6}$	$4.804.10^{-5}$	$8.453.10^{-5}$
72	17	$3.503 \cdot 10^{-1}$	$2.582.10^{-1}$	$7.481.10^{-1}$	$4.527.10^{-1}$
73	17	$8.724.10^{-2}$	$4.419.10^{-2}$	$1.796.10^{-3}$	$1.886.10^{-2}$
74	17	$3.184.10^{-2}$	$1.210.10^{-2}$	$7.433.10^{-4}$	$7.433.10^{-4}$
75	17	$3.811.10^{-2}$	$1.064 \cdot 10^{-2}$	$1.160.10^{-3}$	$1.160.10^{-3}$
76	17	$4.177.10^{-2}$	$4.106.10^{-2}$	$1.565.10^{-1}$	$7.007.10^{-2}$
7 7	17	$1.869.10^{-2}$	$1.035.10^{-2}$	$3.651.10^{-1}$	$9.428.10^{-2}$
78	17	$8.984.10^{-3}$	$2.718.10^{-3}$	$1.120.10^{-3}$	$2.695.10^{-2}$
79	12	$2.253 \cdot 10^{+3}$	$8.890.10^{+2}$	$2.331.10^{+1}$	$7.772.10^{+3}$
80	8	$8.252.10^{-5}$	$6.264.10^{-5}$	$2.702.10^{-3}$	$6.386.10^{-5}$
81	8	$5.410 \cdot 10^{-5}$	$5.416.10^{-5}$	$1.001 \cdot 10^{-3}$	$3.628.10^{-5}$
82	8	$2.502.10^{-4}$	$3.229 \cdot 10^{-4}$	$9.964.10^{-3}$	$2.574.10^{-4}$

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TABLE II

(Continued)

No.	N	Eq. (11)	Eq. (12)	Eq. (13)	Eq. (14)
83	8	$5.398.10^{-4}$	$5.355 \cdot 10^{-4}$	$1.813 \cdot 10^{-2}$	$4.870.10^{-4}$
84	8	$4.726.10^{-5}$	$5.309.10^{-5}$	$4.036.10^{-4}$	$4.838.10^{-5}$
85	8	$4.744.10^{-7}$	$2.971 \cdot 10^{-7}$	$9.518 \cdot 10^{-8}$	$5.178 \cdot 10^{-7}$
86	8	$3.930.10^{-4}$	$1.961 \cdot 10^{-4}$	$1.508 \cdot 10^{-4}$	$4.908 \cdot 10^{-4}$
87	8	$4.822.10^{-6}$	$3.315.10^{-6}$	$4.520.10^{-6}$	$1.226.10^{-5}$
88	8	$2 \cdot 104 \cdot 10^{-6}$	$1.848.10^{-6}$	$1.424.10^{-6}$	$1.983.10^{-6}$
89	11	$8 \cdot 202 \cdot 10^{-3}$	$4.627.10^{-3}$	$1.191.10^{-2}$	$6.460.10^{-3}$
80	11	$5.607.10^{-4}$	$2.328.10^{-4}$	$6.077 \cdot 10^{-3}$	$6.601.10^{-4}$
91	13	$4.788.10^{-1}$	$2.386.10^{-1}$	$2.737.10^{-2}$	$2.737.10^{-2}$
92	13	$4.602.10^{-1}$	$2.334.10^{-1}$	$1.875.10^{-2}$	$1.875.10^{-2}$
93	13	$4.881.10^{-1}$	$2.366.10^{-1}$	$2.766 \cdot 10^{-2}$	$2.766.10^{-2}$
94	13	$4.768.10^{-1}$	$2 \cdot 233 \cdot 10^{-1}$	$3.353.10^{-2}$	$3.353.10^{-2}$
95	17	$9.486.10^{-3}$	$8.459.10^{-3}$	$1.496.10^{-1}$	$7.140.10^{-2}$
96	14	$1.247.10^{-1}$	$2 \cdot 105 \cdot 10^{-2}$	$5.740.10^{-1}$	$2.794.10^{-1}$
97	16	$1.639.10^{-3}$	$1.639.10^{-3}$	$5 \cdot 273 \cdot 10^{-3}$	$5.263.10^{-3}$
98	16	$1.529.10^{-3}$	$1.647.10^{-3}$	$3.797.10^{-2}$	$2.464.10^{-3}$
99	10	$6.710 \cdot 10^{-1}$	$6.254 \cdot 10^{-1}$	$7.382.10^{-1}$	$7.417.10^{-1}$
100	10	$1.386.10^{-2}$	$2.690.10^{-3}$	$1.404 \cdot 10^{-2}$	$1.404.10^{-2}$
101	10	$6.253.10^{0}$	6·444 . 10 ⁰	6·497.10 ⁰	$6.487.10^{0}$
100	10	$1.386.10^{-2}$	$2.690.10^{-3}$	$1.404.10^{-2}$	$1.404.10^{-2}$
102	10	$9.355.10^{-2}$	$7.870.10^{-2}$	$6.103 \cdot 10^{-2}$	$4.817.10^{-2}$
103	10	$8.935.10^{-3}$	$3.724.10^{-3}$	$1.061 \cdot 10^{-2}$	$1.061.10^{-2}$
104	10	$4.127.10^{-4}$	$4.934.10^{-4}$	$1.531.10^{-2}$	$1.943.10^{-3}$
105	11	$1.003.10^{-4}$	$1.117.10^{-4}$	$7.326.10^{-4}$	$6.869.10^{-5}$
106	11	$1.048.10^{-2}$	$1.151.10^{-2}$	$1.566.10^{-2}$	$1.081.10^{-2}$
107	11	$1.024 \cdot 10^{-1}$	$1.148.10^{-1}$	$1.084.10^{-1}$	$1.255.10^{-1}$
108	10	$4.428.10^{-1}$	$4.168.10^{-1}$	$4.950 \cdot 10^{-1}$	$4.888.10^{-1}$
109	10	$4.443.10^{-2}$	$3.624.10^{-2}$	$3.650.10^{-2}$	$3.650.10^{-2}$
110	10	7·975 . 10 ⁰	7·420.10 ⁰	7·742.10 ⁰	$7.742.10^{0}$
111	13	$3.806 \cdot 10^2$	$8.171.10^{1}$	$3.000.10^{0}$	$3.000 \cdot 10^{0}$
112	11	$1.832.10^{1}$	5·047 . 10 ⁰	$2.975.10^{1}$	$2.975.10^{1}$
113	12	$3.845.10^{-3}$	$2.479.10^{-3}$	$1.340.10^{-1}$	$3.591 \cdot 10^{-2}$
114	12	$4.904.10^{-3}$	$2.181 \cdot 10^{-3}$	$4.881.10^{-3}$	$4.881.10^{-3}$
115	11	1·958.10 ⁰	$4.074.10^{-1}$	$2 \cdot 195 \cdot 10^{-2}$	$2.195.10^{-2}$

the experimental error), hence the way of description of the additional Gibbs function is irrelevant. As the non-ideality of behaviour of a mixture is predominantly due to mutual specific interactions of the solvent and solute molecules (formation of hy-

TABLE III

Data			R				
type	Q	Q _{crit}	Eq. (11)	Eq. (12)	Eq. (13)	Eq. (14)	
p <i>K</i>	15.00	7.77	2.63	1.73	2.93	2.70	
IR	16.64	7.76	2.88	1.60	2.86	2.66	
UV-VIS	12.59	7.74	3.00	1.77	2.89	2.34	
NMR	1.32	7.67	2.70	2.10	2.60	2.60	
log k	3.48	7.67	2.70	2.00	3.00	2.30	

Criteria of the Friedman test and average orders of the model equations (11) through (14) for the individual data types in binary mixtures

^{*a*} See ref.¹⁰⁵, significance level $\alpha = 0.05$.

drogen bonds, CT complexes etc.), the conclusions following from Table III agree - in principle - with the ideas about the mechanism of action of solvents in various processes.

The average mean square determined from the mean squares of the regression model (12) applied to logarithms of dissociation constants (in the form of pK) has the value of $5.79 \cdot 10^{-2}$. Hence the average standard deviation is 0.24, which is roughly 2-3 times more than the experimental error in the pK values determined in organic solvents. Although this relation is practically acceptable, it indicates the existence of a non-described behaviour in the system dissociation process – binary solvent. The average mean square determined for log k has the value of $9.70 \cdot 10^{-2}$, which gives the average standard deviation value of 0.31. Also in this case, a part of experimental variability remains unexplained.

From the technical point of view, all the regression models used are well-converging; out of the four-parameter equations (Eqs (11), (13), and (14)), the best-ballanced results are obtained from the 3rd order model by Margules (Eq. (11)) which also shows the fastest convergence at the minimum intercorrelation of parameters. The parameters in Eqs (10) through (12) can advantageously be calculated by means of multiple linear regression.

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